

# Does the hard pomeron obey Regge factorisation?

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## Abstract

While data for the proton structure function demand the presence of a hard-pomeron contribution even at quite small  $Q^2$ , previous fits to the  $pp$  and  $p\bar{p}$  total cross sections have found that in these there is little or no room for such a contribution. We re-analyse the data and show that it may indeed be present and that, further, it probably obeys Regge factorisation:  $\sigma_{\text{HARD}}^{\gamma p}(s, Q_1^2) \sigma_{\text{HARD}}^{\gamma p}(s, Q_2^2) = \sigma_{\text{HARD}}^{pp}(s) \sigma_{\text{HARD}}^{\gamma\gamma}(s, Q_1^2, Q_2^2)$  for all values of  $Q_1^2$  and  $Q_2^2$ .

It has become traditional<sup>[1]</sup> to believe that soft hadronic processes at high energy are dominated by the exchange of a soft pomeron with Regge intercept close to  $1 + \epsilon_1 = 1.08$ , or possibly a little larger<sup>[2]</sup>. Data for the proton structure function  $F_2(x, Q^2)$  have revealed<sup>[3]</sup> also the presence of a hard pomeron, with intercept  $1 + \epsilon_0$  a little greater than 0.4. The hard pomeron is seen particularly clearly in the charm structure function  $F_2^c(x, Q^2)$ , because experiment finds that, while for  $F_2(x, Q^2)$  both hard and soft pomeron contributions are present, the soft pomeron does not contribute to  $F_2^c(x, Q^2)$ .

The data for  $F_2(x, Q^2)$  and  $F_2^c(x, Q^2)$  are consistent with the hard pomeron being a simple Regge pole. One would therefore expect its contributions to different processes to obey Regge factorisation<sup>[4]</sup>. In particular, at each value of  $W = \sqrt{s}$ ,

$$\sigma_{\text{HARD}}^{\gamma\gamma}(s, Q_1^2, Q_2^2) = \frac{\sigma_{\text{HARD}}^{\gamma p}(s, Q_1^2) \sigma_{\text{HARD}}^{\gamma p}(s, Q_2^2)}{\sigma_{\text{HARD}}^{pp}(s)} \quad (1)$$

See figure 1. These relations should hold for all values of  $Q_1^2$  and  $Q_2^2$ , both 0 and nonzero. For the case where one of them is 0, an equivalent statement is

$$F_2^{\gamma \text{ HARD}}(s, Q^2) = \frac{F_2 \text{ HARD}(x, Q^2) \sigma_{\text{HARD}}^{\gamma p}(s)}{\sigma_{\text{HARD}}^{pp}(s)} \quad (2)$$

Our previous strategy was first<sup>[1]</sup> to fit data for purely-hadronic total cross sections with only a soft pomeron and reggeon exchange, that is  $\rho, \omega, f_2, a_2$  exchange. This determined the intercept of the soft pomeron to be close to  $1 + \epsilon_1 = 1.08$ . We fixed this in our subsequent fits<sup>[3]</sup> to the data for the proton structure function  $F_2(x, Q^2)$ , and found that they are described very successfully by adding in a hard pomeron with intercept a little larger than  $1 + \epsilon_0 = 1.4$ . Keeping  $\epsilon_1$  fixed, we then concluded there is little room to include a hard-pomeron term in the purely-hadronic cross sections. We now adopt a different strategy, and simultaneously fit data for  $\sigma^{pp}, \sigma^{p\bar{p}}, \sigma^{\gamma p}$  and the proton structure function  $F_2$ , treating both  $\epsilon_0$  and  $\epsilon_1$  as free parameters.

As is well known, there is a conflict between the measurements of the  $p\bar{p}$  cross section at the Tevatron<sup>[5]</sup>. We assume that there is a better chance of accommodating a hard pomeron in the  $p\bar{p}$  total cross section

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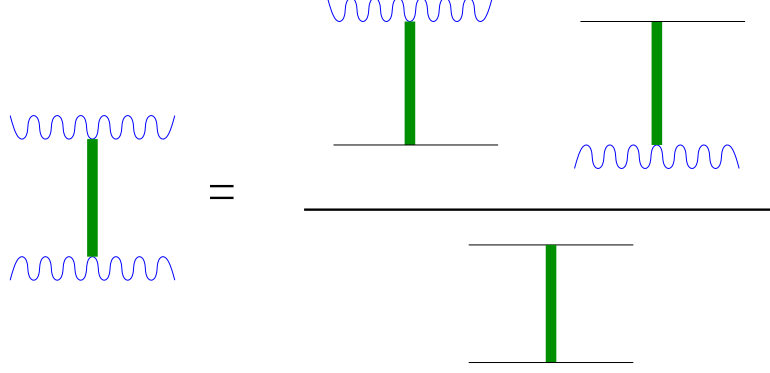


Figure 1: Regge factorisation

if we use the larger CDF measurement at the Tevatron, rather than E710. The details of the fit are dependent on how far down in  $\sqrt{s}$  we wish to extend it. We have chosen to go down to 6 GeV, where previously we chose 10 GeV. After all, we do have extra parameters when we introduce additional, hard-pomeron terms and so should be able to achieve a better fit. Although<sup>[4]</sup> the intercepts of the  $C = +1$  and  $C = -1$  nonpomeron Regge trajectories are not quite equal, we continue to use a single effective intercept  $1 + \epsilon_R$  for them.

So our fits to total cross sections are of the form

$$\sigma = \frac{1}{2p\sqrt{s}} \left( X_0(2\nu)^{1+\epsilon_0} + X_1(2\nu)^{1+\epsilon_1} + Y(2\nu)^{1+\epsilon_R} \right) \quad (3)$$

For the proton structure function  $F_2(x, Q^2)$  at small  $x$  we use

$$F_2(x, Q^2) \sim f_0(Q^2)x^{-\epsilon_0} + f_1(Q^2)x^{-\epsilon_1} + f_R(Q^2)x^{-\epsilon_R} \quad (4a)$$

with

$$\begin{aligned} f_0(Q^2) &= A_0(Q^2)^{1+\epsilon_0} / (1 + Q^2/Q_0^2)^{1+\epsilon_0/2} \\ f_1(Q^2) &= A_1(Q^2)^{1+\epsilon_1} / (1 + Q^2/Q_1^2)^{1+\epsilon_1} \\ f_R(Q^2) &= A_R(Q^2)^{1+\epsilon_R} / (1 + Q^2/Q_R^2)^{1+\epsilon_R} \end{aligned} \quad (4b)$$

We perform a minimum- $\chi^2$  fit to the data for  $\sigma^{pp}$ ,  $\sigma^{p\bar{p}}$ ,  $\sigma^{\gamma p}$  and  $F_2(x, Q^2)$ . (We have not included in our fits the data for  $\rho$ , the ratio of the real and imaginary parts of the forward hadronic amplitudes, because of the theoretical uncertainties in the analysis needed to extract them from experiment.) This leads to the set of parameters

$$\begin{aligned} \epsilon_0 &= 0.452 & \epsilon_1 &= 0.0667 & \epsilon_R &= -0.476 \\ X_0^{pp} &= X_0^{p\bar{p}} = 0.0139 & X_1^{pp} &= X_1^{p\bar{p}} = 24.22 \\ Y^{pp} &= 46.55 & Y^{p\bar{p}} &= 95.81 \\ A_0 &= 0.00151 & Q_0^2 &= 7.85 & A_1 &= 0.658 & Q_1^2 &= 0.600 & A_R &= 1.01 & Q_R^2 &= 0.214 \end{aligned} \quad (5a)$$

so that

$$X_0^{\gamma p} = 0.000169 \quad X_1^{\gamma p} = 0.0737 \quad Y^{\gamma p} = 0.113 \quad (5b)$$

The resulting fits to the  $\gamma p$ ,  $pp$  and  $p\bar{p}$  total cross sections are shown in figures 2a and 2b. They do not look all that different from the old fits without the hard pomeron. Together, the  $\epsilon_0 = 0.45$  hard

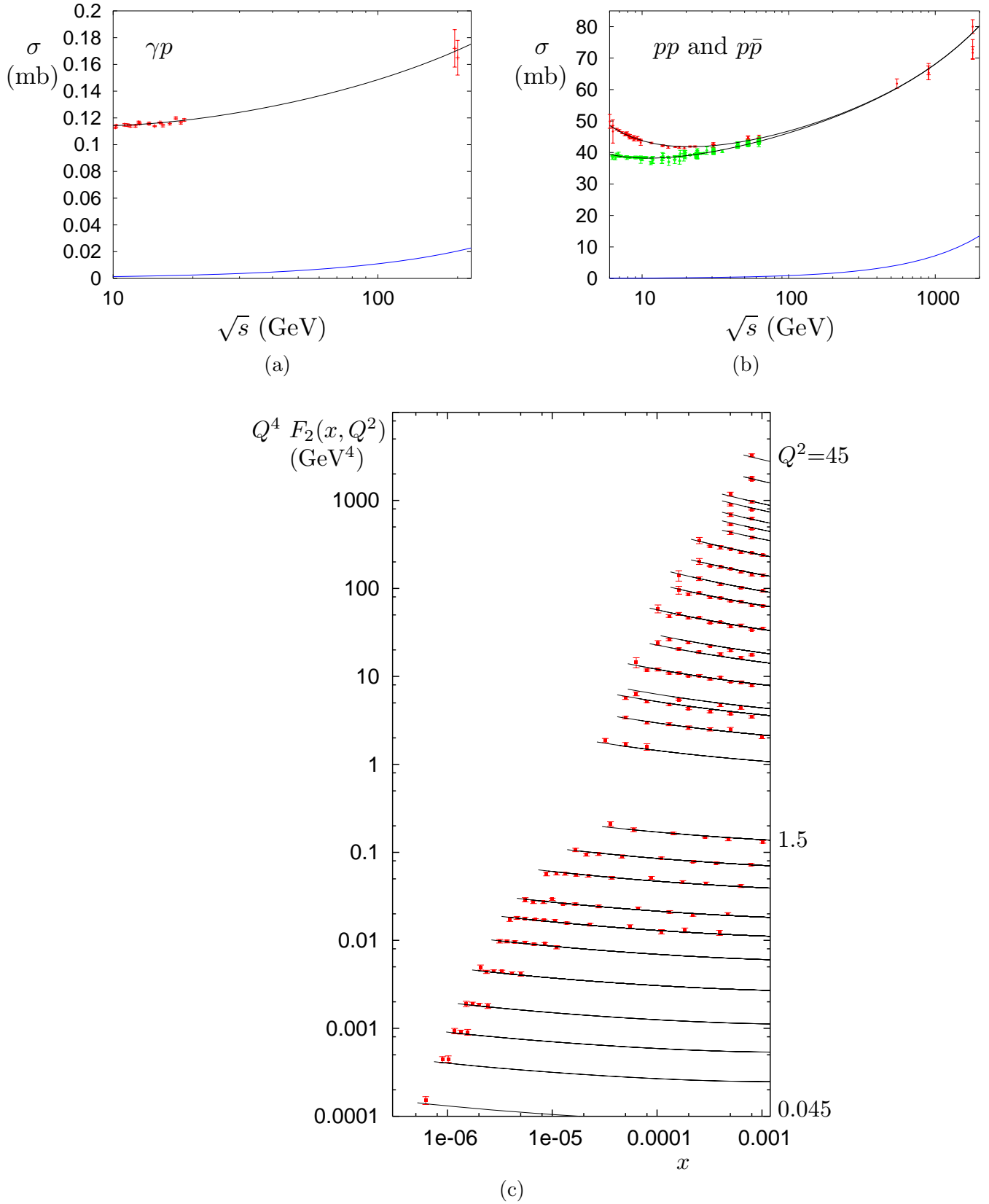


Figure 2: Fits using hard-pomeron, soft-pomeron and reggeon exchange to total cross sections (the lower curves in each plot are the hard-pomeron term) and to HERA data<sup>[6]</sup> for the proton structure function  $F_2(x, Q^2)$ .

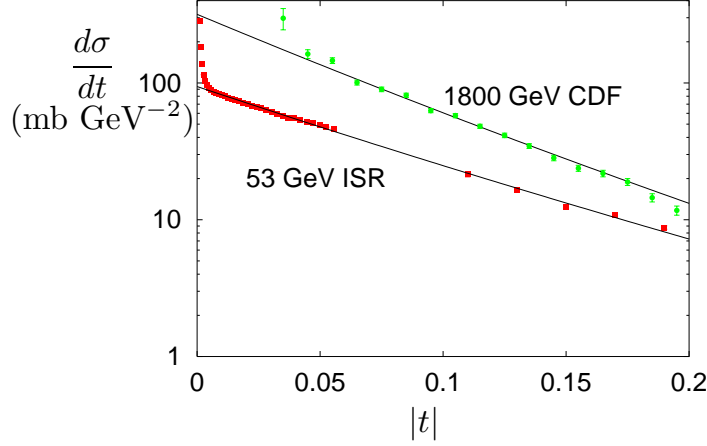


Figure 3:  $pp$  elastic scattering at 53 GeV (CERN ISR<sup>[7]</sup>) and  $p\bar{p}$  at 1800 GeV (CDF<sup>[5]</sup>), with the Regge-theory curves.

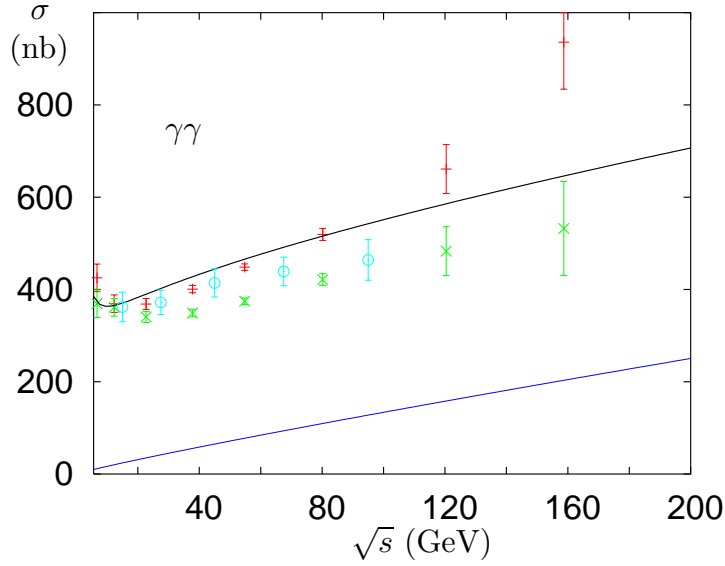


Figure 4: L3 data<sup>[8]</sup> for  $\sigma^{\gamma\gamma}$  using two different Monte Carlos (black points), and OPAL data<sup>[9]</sup> (open points), together with curve obtained from Regge factorisation plus the box graph. The lower curve is the hard-pomeron term.

pomeron and the  $\epsilon_1 = 0.067$  soft pomeron behave not very differently from a single  $\epsilon = 0.08$  pomeron. We may check this by considering  $pp$  and  $p\bar{p}$  elastic scattering. We introduce the Regge trajectories

$$\begin{aligned}
 \alpha_0(t) &= 1 + \epsilon_0 + \alpha'_0 t \\
 \alpha_1(t) &= 1 + \epsilon_1 + \alpha'_1 t \\
 \alpha_R(t) &= 1 + \epsilon_R + \alpha'_R t
 \end{aligned} \tag{6}$$

There is evidence from  $J/\psi$  photoproduction that  $\alpha'_0$  is small<sup>[15]</sup>. If  $\alpha_0(t)$  and  $\alpha_1(t)$  together resemble the single canonical trajectory with slope  $\alpha' = 0.25$  (in GeV units), we need  $\alpha'_1$  to be greater than

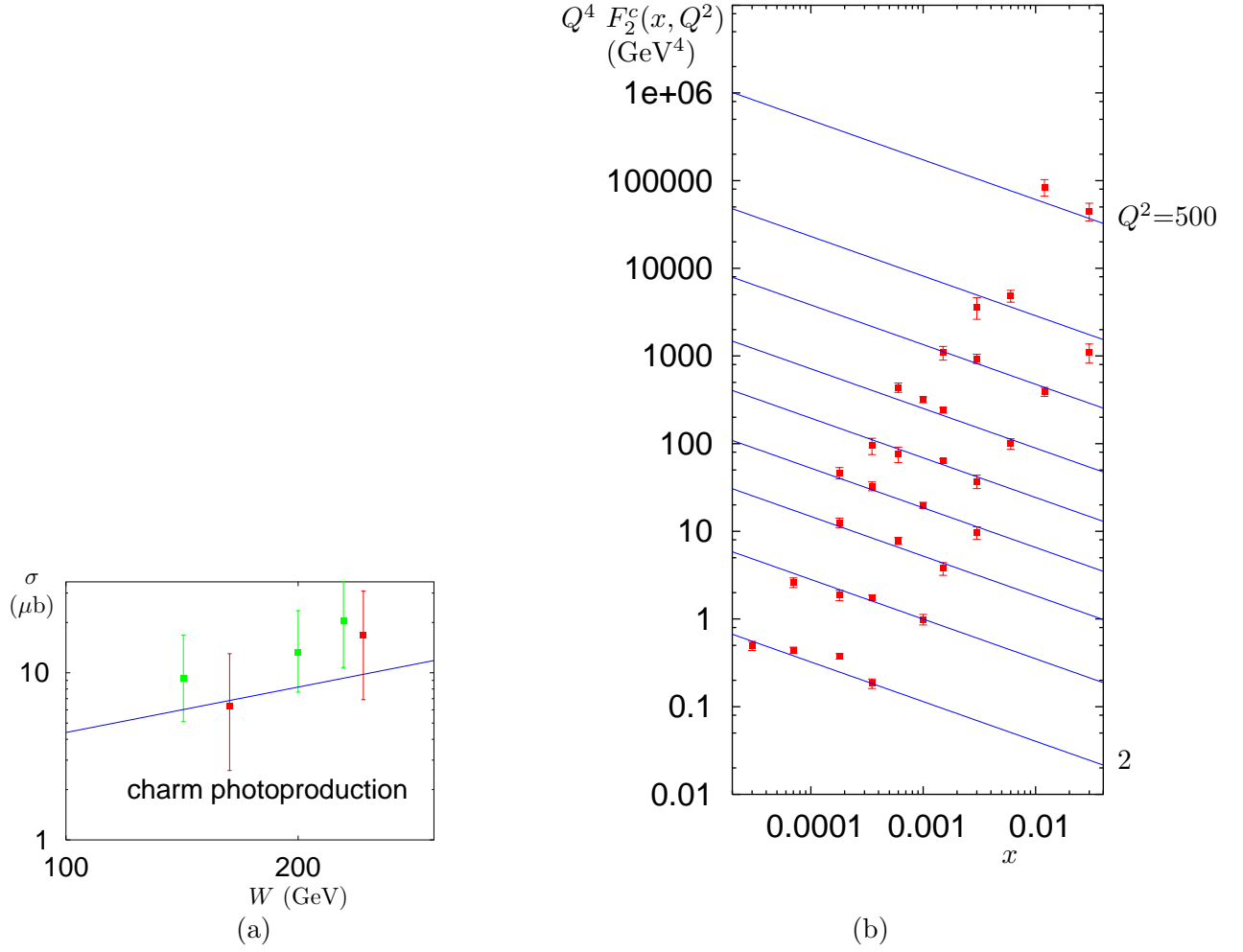


Figure 5: The charm photoproduction cross section, and ZEUS data<sup>[10]</sup> for the charm structure function  $F_2^c(x, Q^2)$  and The curves are 0.4 times the hard-pomeron contributions to the curves in figures 2a and 2c

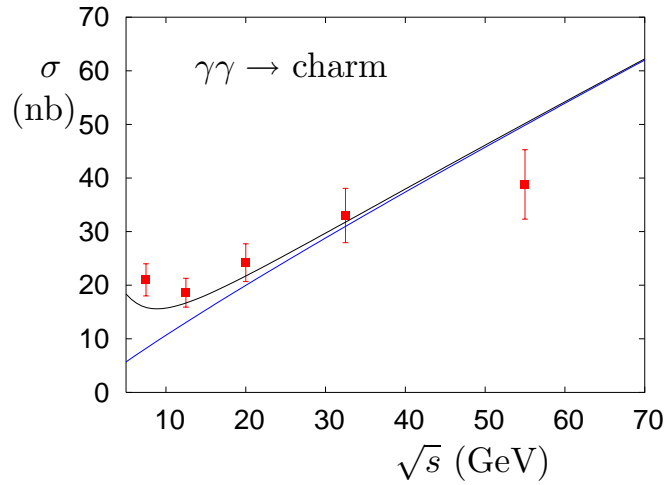


Figure 6: L3 data<sup>[11]</sup> for  $\gamma\gamma \rightarrow \text{charm}$ . The lower curve is the hard-pomeron contribution obtained from Regge factorisation; the upper curve includes also the box graph.

0.25. We choose

$$\alpha'_0 = 0.1 \qquad \alpha'_1 = 0.3 \qquad (7)$$

though these values are open to some adjustment. The resulting differential cross sections are shown in figure 3. Note that we have plotted the CDF data at  $\sqrt{s} = 1800$  GeV rather than E710, because our fit to the total cross section favoured CDF rather than E710. As in the past<sup>[16][4]</sup>, we have coupled the pomerons to the proton through the Dirac elastic form factor  $F_1(t)$ .

If we apply the factorisation (1), apply it similarly to soft-pomeron and regge exchange, and add in a contribution from the simple box graph, we obtain the  $\gamma\gamma$  total cross section shown in figure 4. The data need large acceptance corrections and so are highly sensitive to the Monte Carlo used for this. The L3 data<sup>[8]</sup> shown are the outputs from two of their Monte Carlos.

Regge factorisation should apply also to cross sections for charm production. We have already observed<sup>[3][17]</sup> that the data for the charm structure function<sup>[10]</sup>  $F_2^c$  show that at small  $x$  it is described by hard-pomeron exchange alone. Further, it is given by 0.4 times the hard-pomeron component of  $F_2$ . That is, the hard pomeron's couplings to  $u, d, s, c$  quarks are equal. This remains true even down to  $Q^2 = 0$ ; see figure 5. Combining the flavour-blindness of the hard-pomeron coupling with Regge factorisation, we conclude that the hard-pomeron contribution to the cross section for  $\gamma\gamma \rightarrow$  charm should be  $0.64 [\sigma_c^{\gamma p}]^2 / \sigma_{\text{HARD}}^{pp}$ . This corresponds to the lower curve in figure 6. Adding in the box graph, with  $m_c = 1.3$  GeV, gives the upper curve.

As we have said, the factorisation should apply also at nonzero  $Q^2$ . We apply it as in (2), and treat similarly soft-pomeron and reggeon exchange. Ideally, we should keep to  $x < 0.001$ , as in figure 2c, as for larger  $x$  we need to include some factor of  $x$  which is equal to 1 at small  $x$  but  $\rightarrow 0$  as  $x \rightarrow 1$ . This factor is unknown, so we simply use a power of  $(1-x)$  determined by the old dimensional counting rules<sup>[18]</sup>. This leads us to include a factor  $(1-x)^5$  in each of the pomeron contributions, and  $(1-x)$  in the reggeon contribution. These factors are not correct, as they take no account of perturbative evolution, but it is better to include them than not to do so. Adding in the box graph gives us the results for the photon structure function  $F_2^\gamma(x, Q^2)$  shown in figure 7. Again, in each case the lower curve is the hard-pomeron component. It seems that the OPAL data at  $Q^2 = 10.7$  and  $17.8$  GeV<sup>2</sup> may be out of line with the other data. This impression is enhanced if we plot data at values of  $x$  close to 0.01 against  $Q^2$ , as in figure 8. Figure 9 shows the prediction for the photon's charm structure function at  $Q^2 = 20$  GeV<sup>2</sup>, together with the data point from OPAL<sup>[13]</sup>.

Lastly, we consider the cross section for  $\gamma\gamma \rightarrow$  hadrons when both photons are off shell: see figure 9.

We are not able to reach a firm conclusion about whether the hard pomeron obeys Regge factorisation, because the tests involve data that require large Monte Carlo acceptance corrections and are therefore subject to no little uncertainty. We have applied several tests:  $\sigma^{\gamma\gamma}, \sigma_c^{\gamma\gamma}, F_2^\gamma$  and  $\sigma^{\gamma^*\gamma^*}$ . None of them clearly fails.

Note that in any case factorisation should not be exact. The hard and soft pomeron contributions are not really powers of  $s$  or  $1/x$ . The powers we have used should be regarded as effective powers. Assuming, as we have, that each pomeron yields a simple power  $e_0$  or  $e_1$ , then there are corrections from the exchange of two or more pomerons. These are negative and so result in effective powers  $\epsilon_0 < e_0$  and  $\epsilon_1 < e_1$ . If factorisation is approximately satisfied, this is an indication that the corrections are small, so that  $e_0$  is only a little greater than  $\epsilon_0$  and  $e_1$  is a little greater than  $\epsilon_1$ . When we analysed<sup>[19]</sup>  $pp$  elastic scattering data using only the soft pomeron, we concluded that the correction term was at the 10% level for the total cross section at Tevatron energy. It will surely be higher at LHC energy, so that the hard-pomeron effective power will be reduced at this energy. Without such a reduction, the cross section at LHC energy would extrapolate to 165 mb. So, while we do not believe that it will be

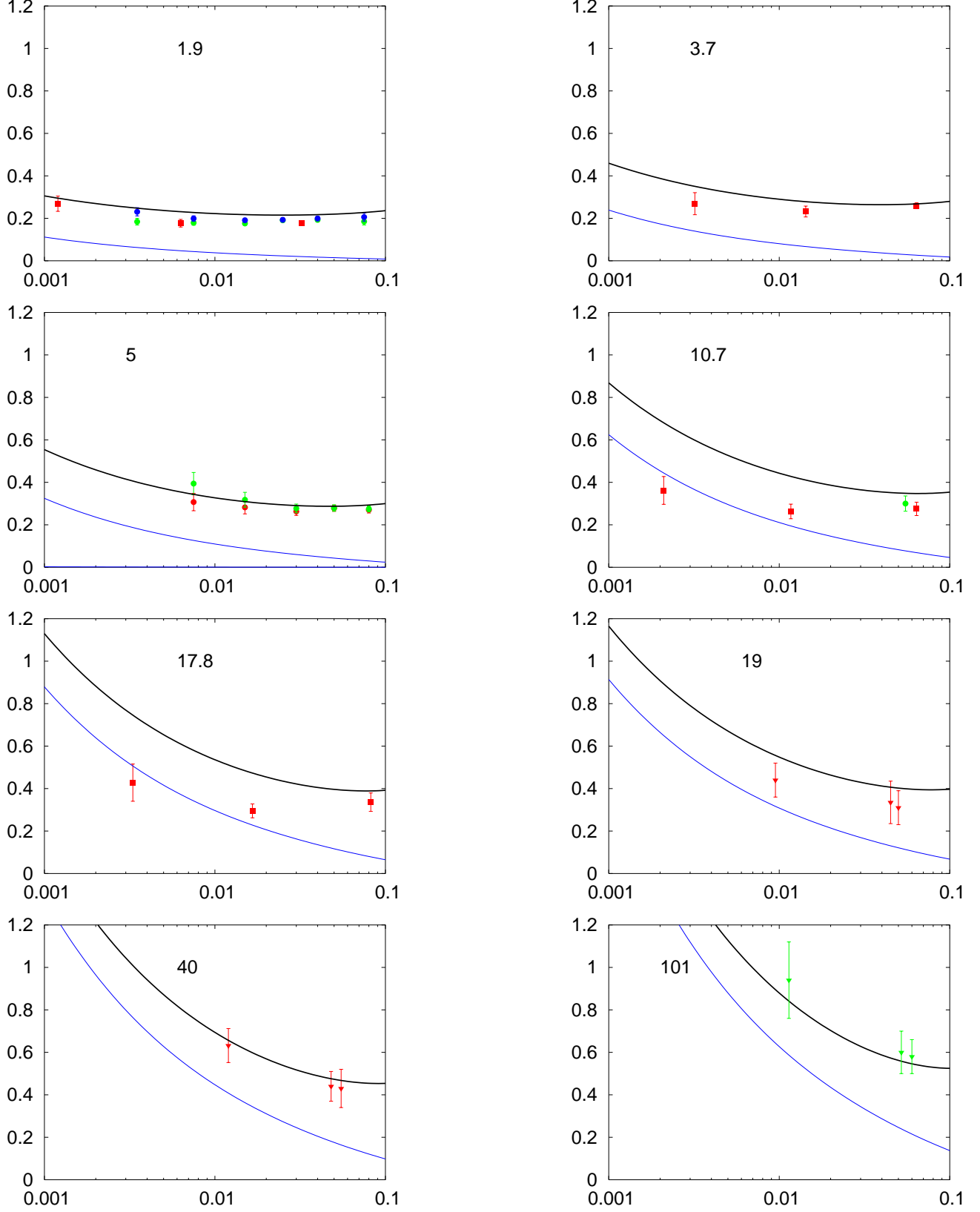


Figure 7: Data<sup>[12]</sup> the photon structure function  $F_2^\gamma/\alpha$  plotted against  $x$  at various values of  $Q^2$  with fits from Regge factorisation. In each case the lower curve is the hard-pomeron contribution.

as large as this, there is a real prospect that the cross section will be found to be significantly larger than the prediction of 101 mb given by our old data fit<sup>[1]</sup> fit based on the soft pomeron alone.

Related analyses to this one have been performed by Donnachie and Dosch<sup>[20]</sup> and by Cudell and collaborators<sup>[21]</sup>, though the details are very different. The former is based on a dipole picture, but gives results similar to ours, while the latter maintains that corrections from multiple exchanges are large.

Finally, we note that, as with our previous analysis<sup>[22]</sup>, as soon as  $Q^2$  is large enough for the DGLAP equation to be valid, the variation of the hard-pomeron coefficient function  $f_0(Q^2)$  we have extracted from the data and given in (4) and (5), agrees exactly with the predictions of LO evolution. We show this in figure 10. As before<sup>[22]</sup>, NLO evolution gives almost the same result.

*This research was supported in part by PPARC*



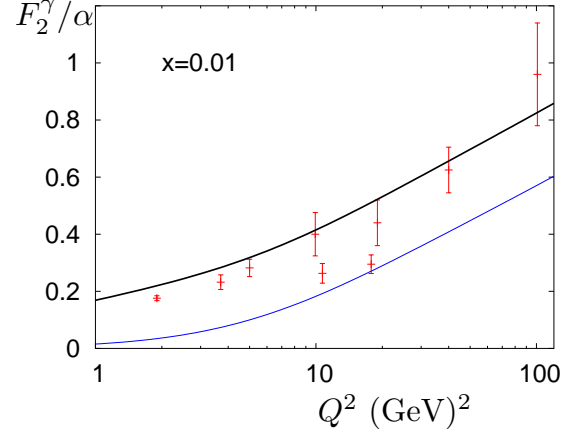


Figure 8: Data<sup>[12]</sup> for the photon structure function  $F_2^\gamma/\alpha$  at  $x$  close to 0.01 plotted against  $Q^2$ , with the prediction from Regge factorisation. The lower curve corresponds to hard-pomeron exchange alone.

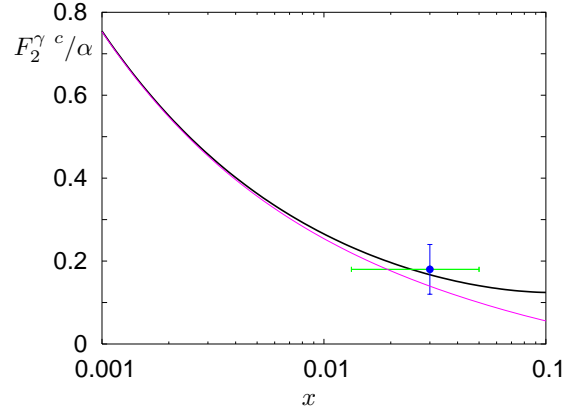


Figure 9: Data point<sup>[13]</sup> for the photon structure function  $F_2^{\gamma c}/\alpha$  at  $Q^2 = 20 \text{ GeV}^2$  plotted against  $Q^2$ , with the prediction from Regge factorisation. The lower curve corresponds to hard-pomeron exchange alone.

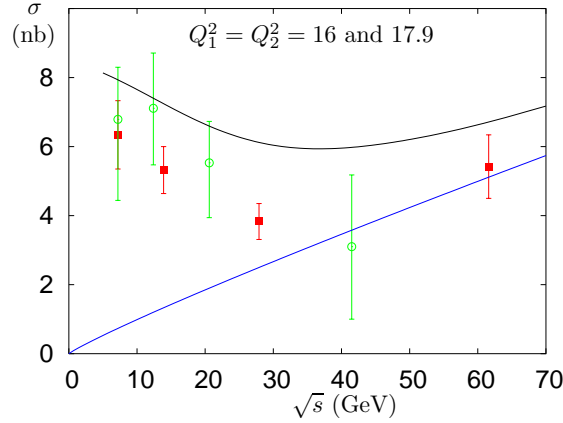


Figure 10: L3 and OPAL data<sup>[14]</sup> for  $\sigma(\gamma^*\gamma^*)(W)$  at  $Q^2 = 16$  and  $17.9 \text{ GeV}^2$ . The lower curve corresponds to hard-pomeron exchange alone.

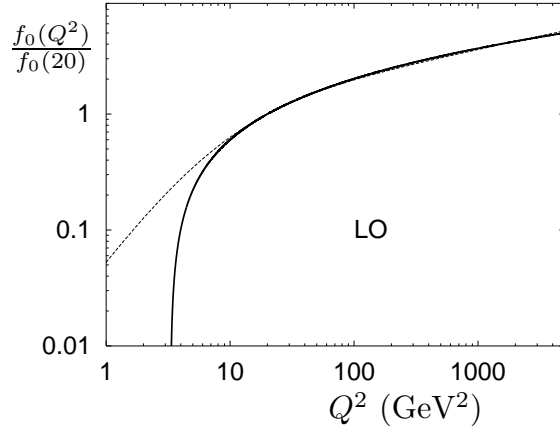


Figure 10:  $Q^2$  evolution of the hard-pomeron coefficient function. The upper curve is extracted from the data, and the lower corresponds to LO DGLAP evolution.

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